Theoretical Cosmology Exam

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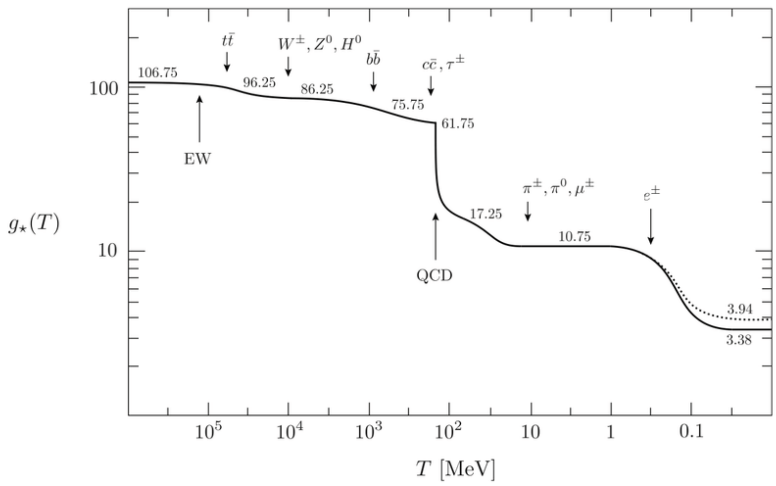
The exam is divided in two parts: **theory questions** (60 points) and **problems** (50 points). You need at least 50 points to pass the exam. The exam can be answered in English or German.

1 Theory questions

The following questions must be answered **shortly**, i.e. using few sentences.
Each question is worth *6 points*.

(a) Indicate if the *particle horizon* and the *event horizon* in a flat FLRW metric are finite or infinite, when the Universe is filled only by 1) radiation, 2) matter, 3) a cosmological constant.

(b) When measuring distances in cosmology, what is the advantage of using the *luminosity distance* or the *angular distance*, instead of the *metric distance*?



(c) Explain why the effective number of relativistic degrees of freedom $g_*(T)$ (see figure above) decreases with the temperature.

(d) A given particle has an effective interaction rate Γ with the thermal cosmic plasma. What condition must Γ obey, for the particle to be coupled to the plasma?

(e) How could a *weak interacting massive particle* (WIMP) solve the dark matter problem?

(f) Bring the following cosmological events in the right order: BBN, recombination, inflation, matter-radiation equality ($\rho_{\rm m} = \rho_{\rm rad}$), reheating, matter-dark energy equality ($\rho_{\rm m} = \rho_\Lambda$).

(g) In the SVT decomposition of the spacetime metric, how many scalar, vector, and tensor degrees of freedom are there? How many of them are physical?

(Turn over the page)

(h) Indicate two properties of the *comoving curvature perturbation* $\mathcal{R}$ that make it a useful variable in cosmology.

(i) What is the most important contribution to the dipole anisotropy in the Cosmic Microwave Background? Explain it.

(j) The equations of motion for a homogeneous scalar field ϕ with potential energy $V(\phi)$ evolving in a flat FLRW spacetime are

$$H^2 = \frac{1}{3M_{\rm pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) , \quad \ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 , \tag{1}$$

where $H \equiv H(t)$ is the Hubble parameter. Write these equations in the slow-roll approximation by neglecting the appropriate terms.

2 Problems

<u>Problem 2.1:</u> Friedmann equations for a flat Universe *(25 points).*
Consider the Friedmann equations for a spatially-flat FLRW Universe,

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho , \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) . \tag{2}$$

(a) Show that these equations necessarily imply the following conservation constraint,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 . \tag{3}$$

(b) Using Eq. (3), show that the energy density of a fluid with constant equation of state $w \equiv p/\rho$ evolves as $\rho \propto a^{-3(1+w)}$. Particularize this result for matter, radiation, and dark energy.

(c) Show that the age of a Universe filled only with matter is $t_0 = 2/(3H_0)$, where H_0 is the Hubble parameter today.

<u>Problem 2.2:</u> Inflation with quartic potential *(25 points).*
Consider the following quartic inflationary potential,

$$V(\phi) = \frac{\lambda}{4}\phi^4 , \tag{4}$$

where ϕ is the inflaton field, and $\lambda > 0$.

(a) For which values of ϕ can slow-roll inflation happen?

(b) What are the slow-roll predictions for the spectral index n_s and the tensor-to-scalar ratio r in this model? [use $N_* = 60$]. Is this model compatible with current experimental observations?

(c) Which value of λ does one need to generate the observed amplitude $A_s = 2.3 \times 10^{-9}$ of scalar perturbations?

Hint: Recall that

$$\epsilon_{\rm V} = \frac{m_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2 , \quad \eta_{\rm V} = m_{\rm Pl}^2\frac{V''}{V}, \quad N(\phi) = \int \frac{|d\phi|}{\sqrt{2m_{\rm Pl}^2\epsilon_{\rm V}(\phi)}} \tag{5}$$

$$r = 16\epsilon_{\rm V*}, \quad n_s = 1 - 6\epsilon_{\rm V*} + 2\eta_{\rm V*}, \quad A_s = \frac{V_*}{24\pi^2 m_{\rm Pl}^4\epsilon_{\rm V*}} \tag{6}$$