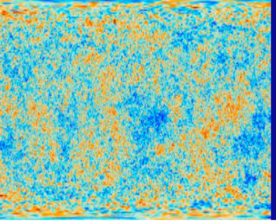
Theoretical cosmology

Introduction

Francisco Torrentí
f.torrenti@unibas.ch

Master in Physics - Frühjahrsemester 2021



Basic information

- **Theory**: Each Wednesday from 14:00 - 16:00 h (~14 lectures)

  Lecturer: Francisco Torrenti

  f.torrenti@unibas.ch - Skype: tsocap - Office: 4.15, PH

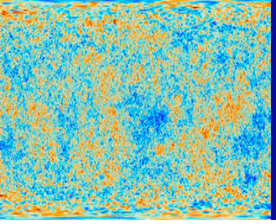
- **Exercises**: Approx. every two weeks, starting the week 15.03-19.03 [Date to be decided] (~6 sheets)

  Lecturer: Kenneth Marschall

  kenneth.marschall@unibas.ch - Skype: ken.marschall - Office: 4.15, PH

> **LINK TO ZOOM:** https://unibas.zoom.us/j/94958214553?pwd=RXdpUWcxWTJKd3NLSXBzQ0JHbnJLdz09

- **Exam**: Wednesday 9th of June, 14:00 - 16:00 h



Bibliography

PREREQUISITES: General relativity, Quantum Field Theory

BOOKS:

- Baumann lectures on cosmology

  http://cosmology.amsterdam/education/cosmology/

- *Physical Foundations on cosmology*, V. Mukhanov.
- *Modern cosmology*, S. Dodelson.
- *Cosmology*, S. Weinberg.
- *Primordial cosmology*, P. Peter, J-P. Uzan.
- *Spacetime and geometry*, S. M. Carroll.

Contents of the course

1. **Geometry and Dynamics of the Universe**

   FLRW metric, distances, Friedmann equations…

2. **Thermal History of the Universe**

   Equilibrium distributions, energy and entropy, Boltzmann equations, CDM, recombination, BBN…

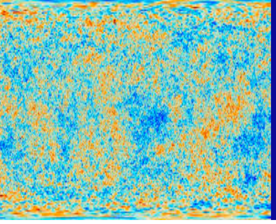
3. **Cosmological Perturbation Theory and structure formation**

   Perturbed equations, initial conditions, curvature perturbation, DM clustering, CMB…

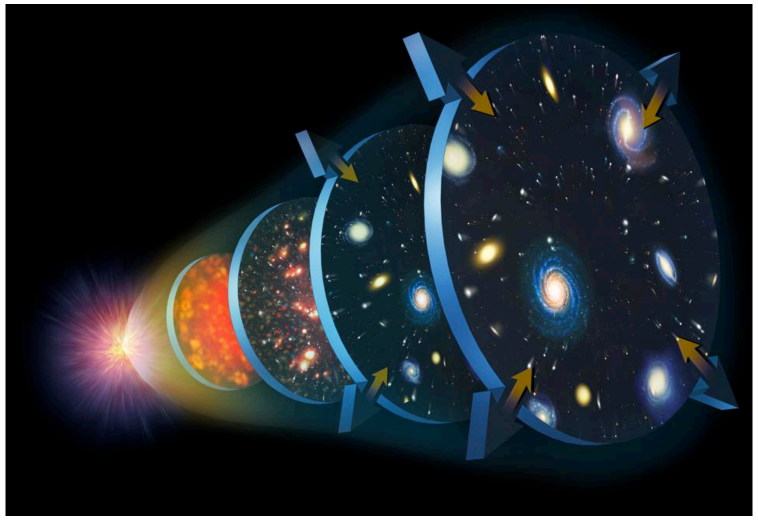
4. **Inflation**

   Homogenity and flatness problems, slow-roll inflation, quantum fluctuations, reheating…

COSMOLOGY: an overview on the contents of the course



What is cosmology?



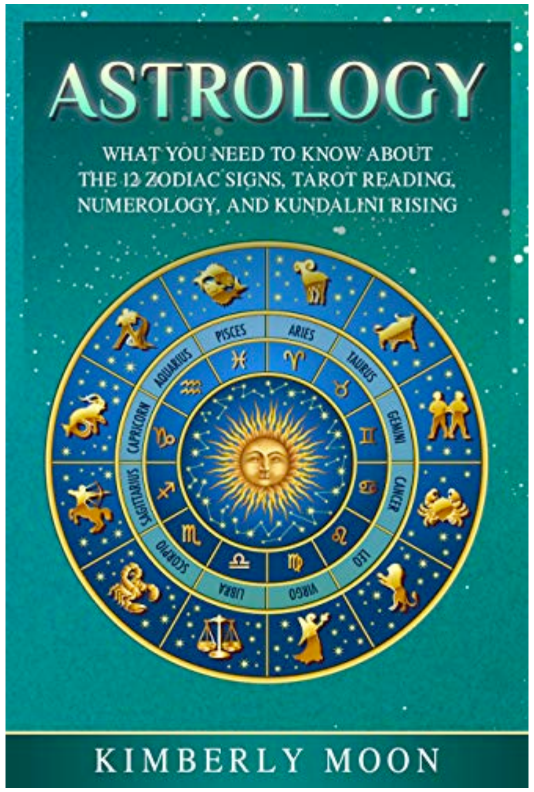
- **COSMOLOGY:**
  - *The scientific study of the origin, evolution, and large scale structure of the universe.*
  - *Treats the universe as a whole*

- **COSMOLOGY is NOT:**

Astronomy:
Studies individual objects (eg. stars, planets, galaxies), normally with observations

Astrophysics:
Uses laws of physics to explain the nature of those objects

Astrology:
Pseudoscience



Modern cosmology

MODERN COSMOLOGY
based on

(HOT) BIG BANG MODEL

- Describes the evolution of the Universe from a fraction of a second until today

INFLATION

- Provides the initial conditions for the hot Big Bang model

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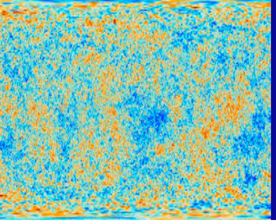
Cosmological principle

<u>COSMOLOGICAL PRINCIPLE</u>:
The Universe is homogeneous and isotropic at large scales.

HOMOGENEITY
is a **global property**:
there are no preferred points in space

ISOTROPY
is a **local property**:
Universe looks the same at all directions



The FLRW metric

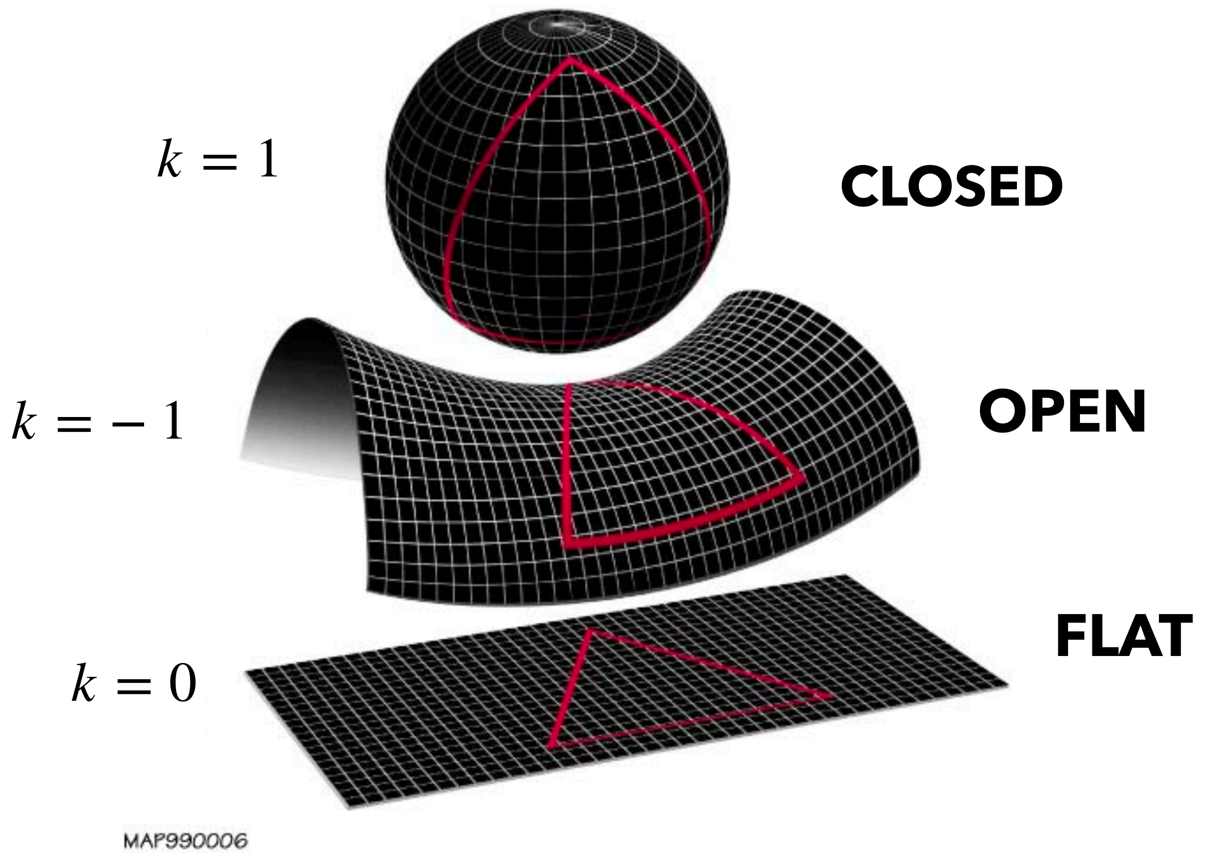
The only 4D metric compatible with the cosmological principle is the FRIEDMANN-LEMAITRE-ROBERTSON-WALKER (FLRW) METRIC

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

a(t): scale factor

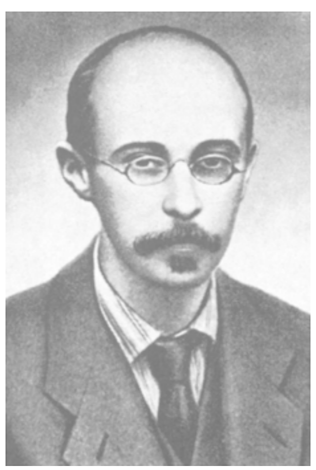
It gives the size of the spatial slices, and depends on time: the Universe **expands or contracts**

k: spatial curvature



The FLRW metric

- The main results of the FLRW models where derived by Friedmann (1925) and, independently, by Lemâitre (1927).
- Robertson and Walker proved rigorously that the FLRW metric is the only homogeneous and spatially isotropic metric (1935).



A. Friedmann

G. Lemâitre

H. Robertson

A. Walker

- FLRW
- FRW
- RW
- FL

https://en.wikipedia.org/wiki/Friedmann–Lemaître–Robertson–Walker_metric

The Friedmann equations

Cosmological principle + General Relativity

↓

FRIEDMANN EQUATIONS

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p + \Lambda)$$

ρ(t): energy density

p(t): pressure

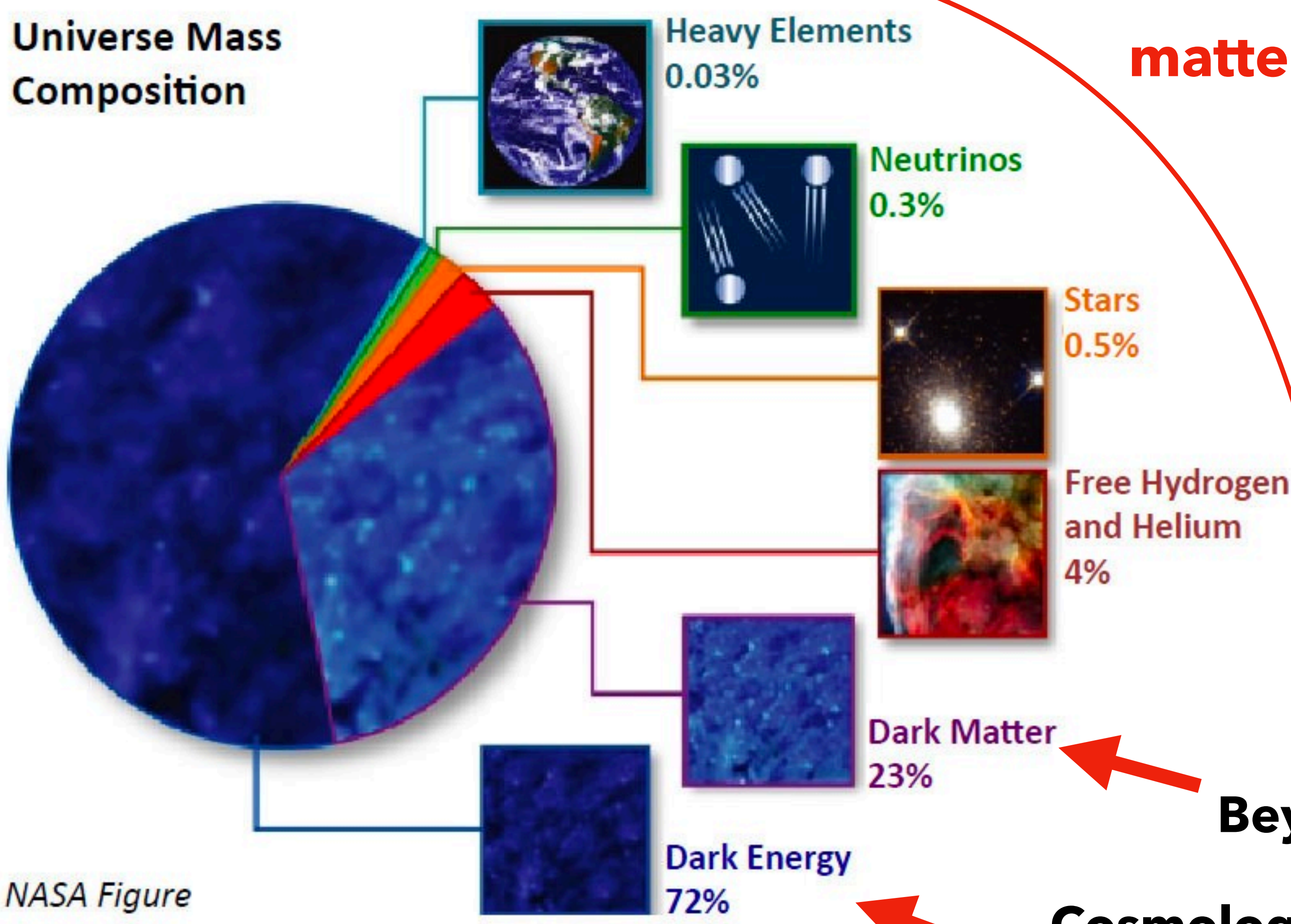
k: spatial curvature

Λ: cosmological constant

EXPANSION / CONTRACTION ⟷ **CONTENT OF THE UNIVERSE**

ΛCDM model: parametrization of the Big Bang

Ordinary matter



NASA Figure

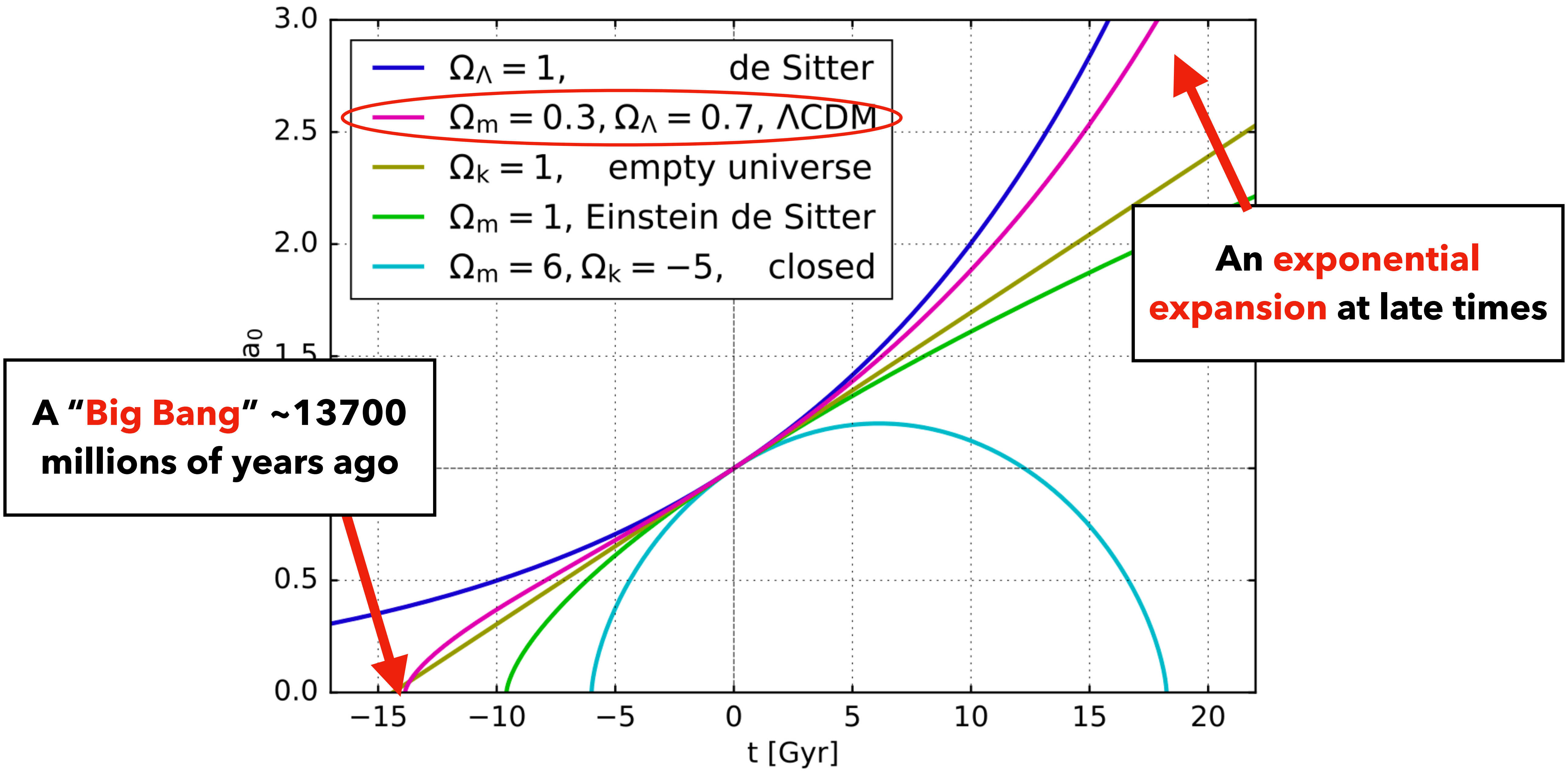
We only understand 4% of the content of the Universe!

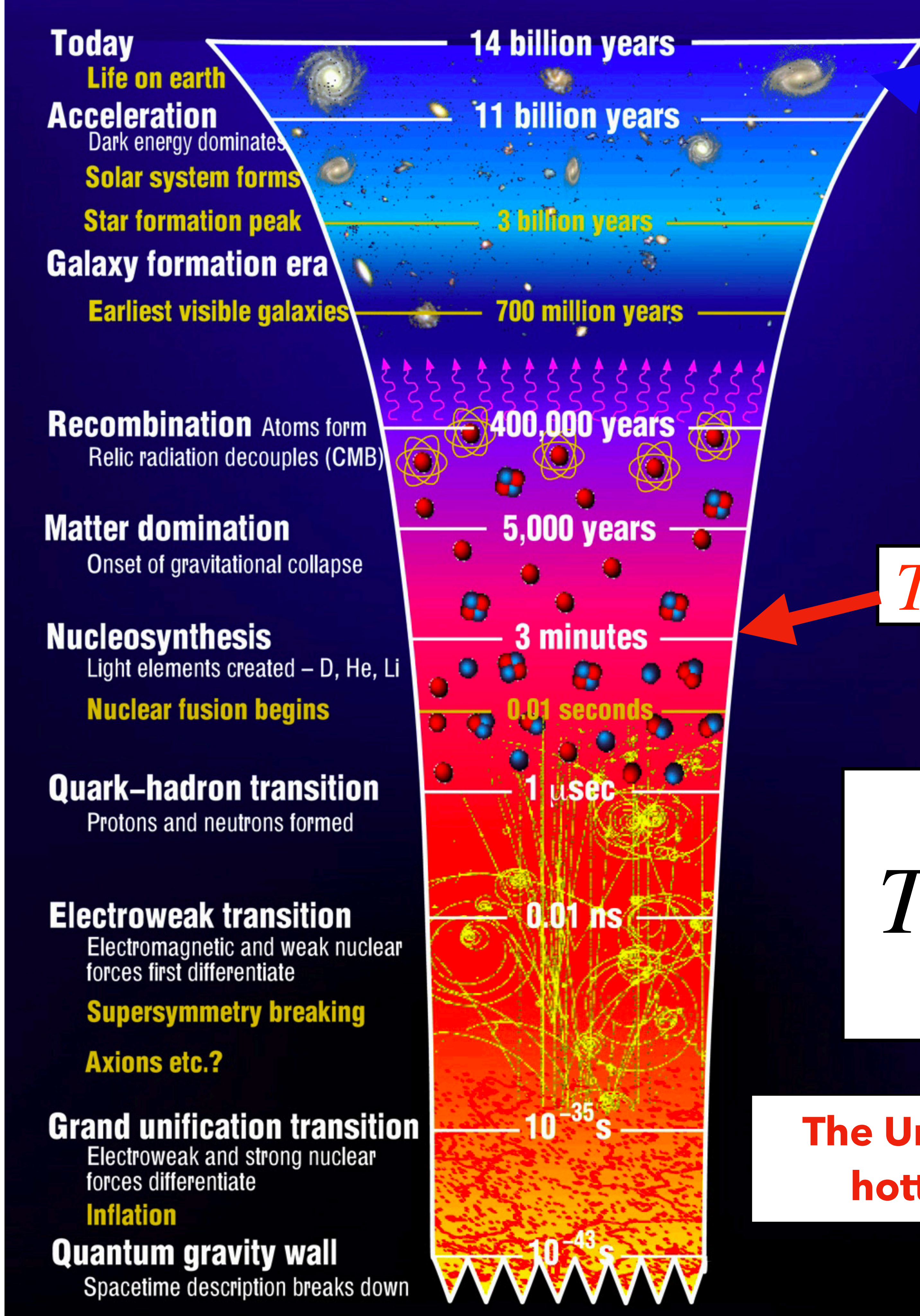
Beyond-the-SM particle?

Cosmological constant (vacuum energy?)

The ΛCDM MODEL

$$\Omega_m \equiv \frac{8\pi G}{3H_0^2}\rho_m \qquad \Omega_\Lambda \equiv \frac{\Lambda}{3} \qquad \Omega_k \equiv \frac{-k}{a_0^2 H_0^2} \quad \Rightarrow \quad \boxed{\Omega_m + \Omega_k + \Omega_\Lambda = 1}$$





$T \approx 2.73\mathrm{K}$

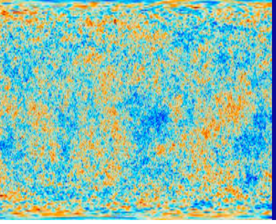
$T \approx 10^9\mathrm{K}$

$$T \propto \frac{1}{a(t)}$$

The Universe was much hotter in the past!!

Observational evidences of the Big Bang theory

1. Expansion of the Universe
2. Abundances of light elements
3. Cosmic Microwave Background
4. Formation of galaxies and large-scale structure



Big Bang theory

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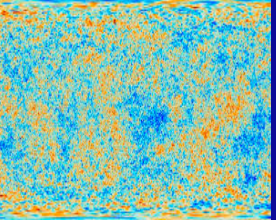


Expansion of the Universe

FLRW metric → **HUBBLE LAW:** $v = H_0 d$

"Effective velocity" inferred by the expansion

Results from Hubble (1929):

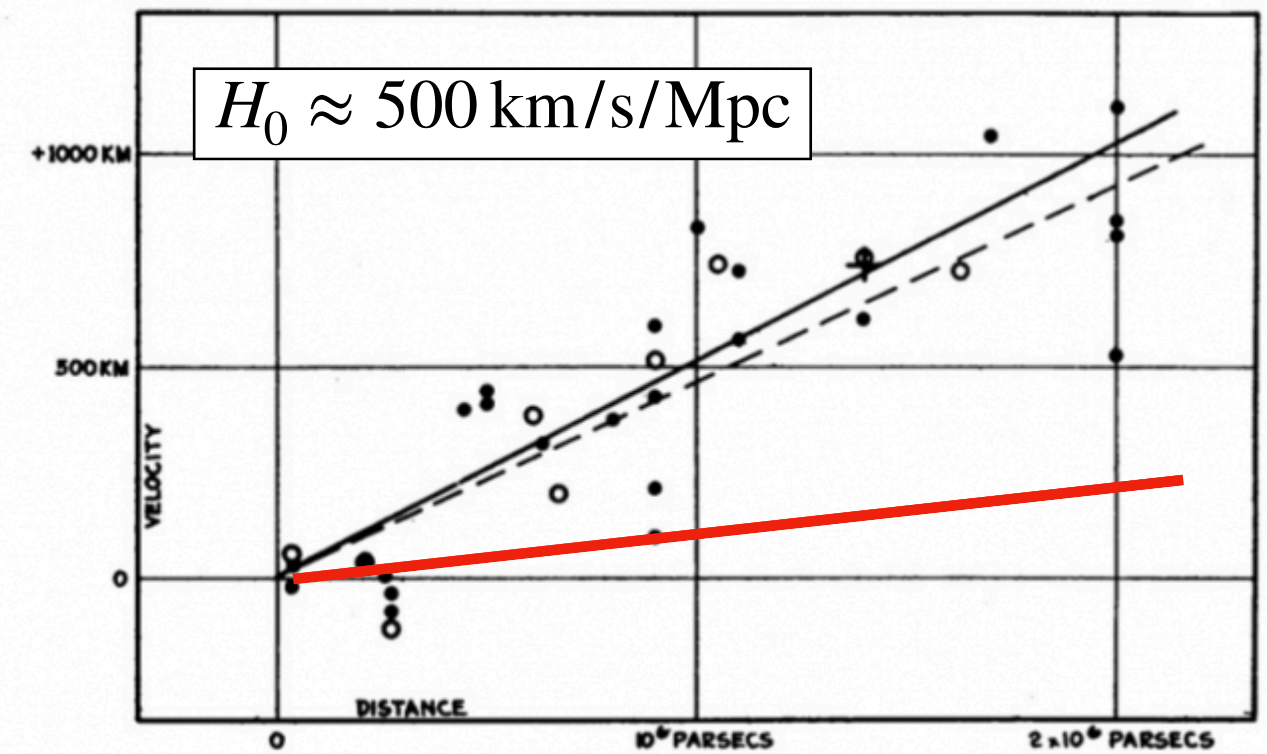
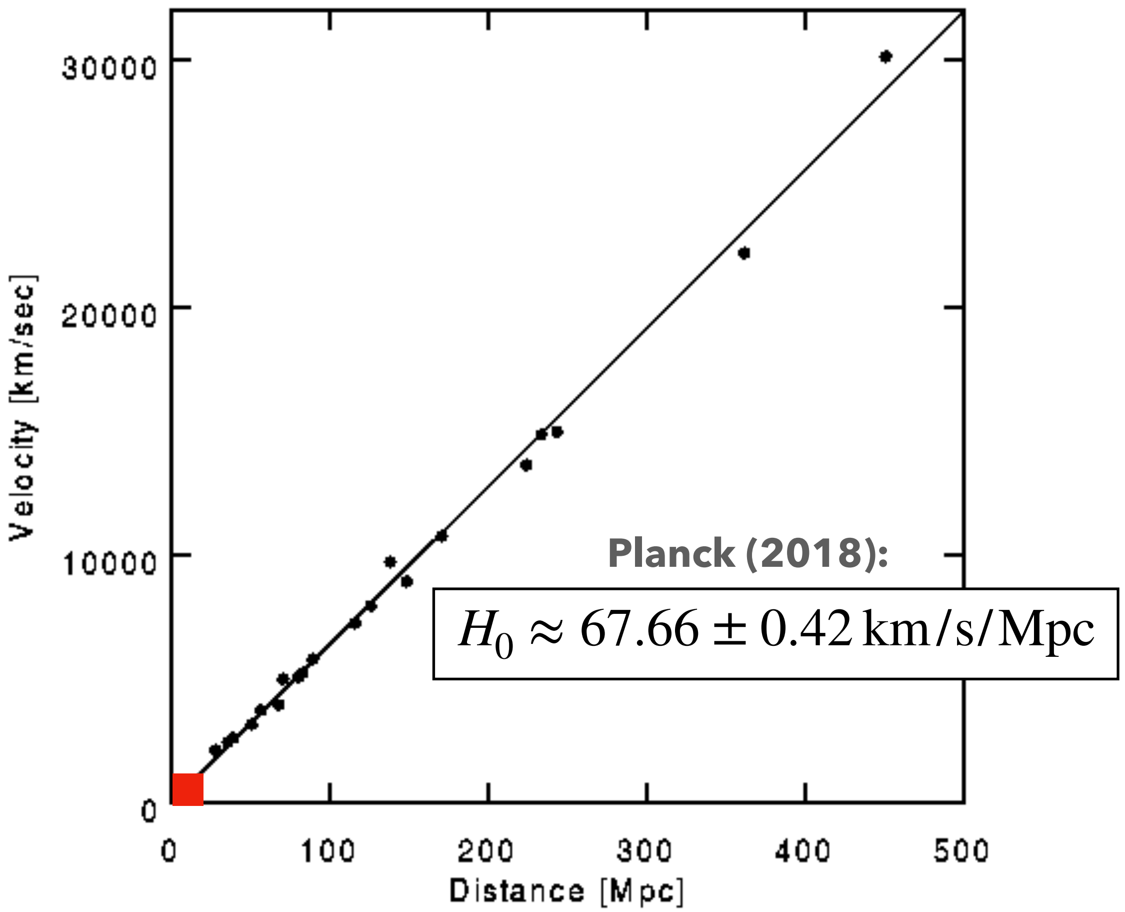


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

Modern data:



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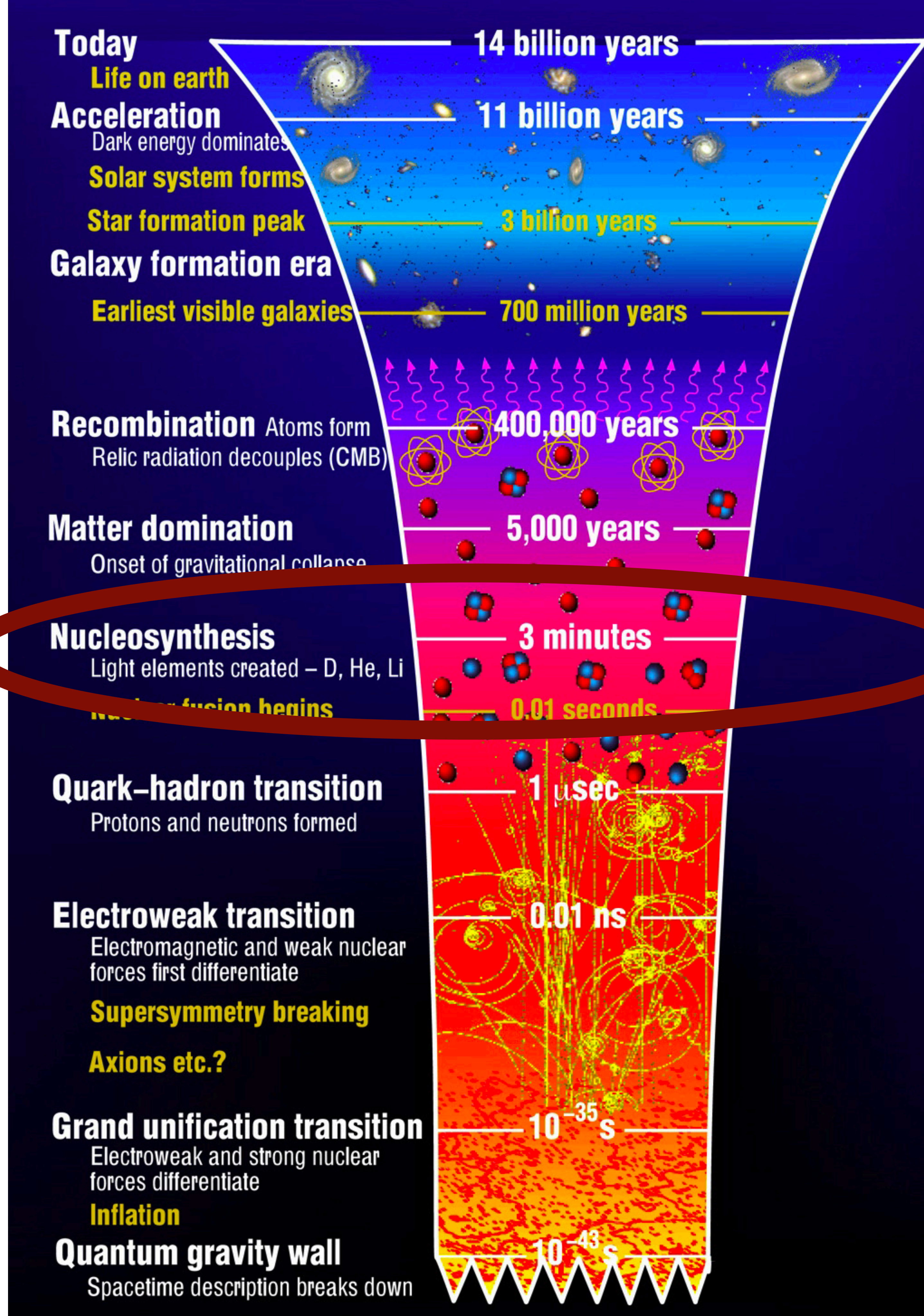

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14 billion years
Life on earth
Acceleration
11 billion years
Dark energy dominates
Solar system forms
Star formation peak
3 billion years
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10^-35 s
Electroweak and strong nuclear forces differentiate
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Quantum gravity wall
10^-43 s
Spacetime description breaks down

Big Bang nucleosynthesis (BBN)

- **At T ≈ 10^9 K (t ≈ 3 min), the temperature is small enough that new atomic nucleons start forming from protons and neutrons.**

- **Chain of nuclear reactions:**

$n^0 \longrightarrow p^+ + e^- + \overline{\nu}_e$	$p^+ + n^0 \longrightarrow {}^2_1\mathrm{D} + \gamma$
${}^2_1\mathrm{D} + p^+ \longrightarrow {}^3_2\mathrm{He} + \gamma$	${}^2_1\mathrm{D} + {}^2_1\mathrm{D} \longrightarrow {}^3_2\mathrm{He} + n^0$
${}^2_1\mathrm{D} + {}^2_1\mathrm{D} \longrightarrow {}^3_1\mathrm{T} + p^+$	${}^3_1\mathrm{T} + {}^2_1\mathrm{D} \longrightarrow {}^4_2\mathrm{He} + n^0$
${}^3_1\mathrm{T} + {}^4_2\mathrm{He} \longrightarrow {}^7_3\mathrm{Li} + \gamma$	${}^3_2\mathrm{He} + n^0 \longrightarrow {}^3_1\mathrm{T} + p^+$
${}^3_2\mathrm{He} + {}^2_1\mathrm{D} \longrightarrow {}^4_2\mathrm{He} + p^+$	${}^3_2\mathrm{He} + {}^4_2\mathrm{He} \longrightarrow {}^7_4\mathrm{Be} + \gamma$
${}^7_3\mathrm{Li} + p^+ \longrightarrow {}^4_2\mathrm{He} + {}^4_2\mathrm{He}$	${}^7_4\mathrm{Be} + n^0 \longrightarrow {}^7_3\mathrm{Li} + p^+$

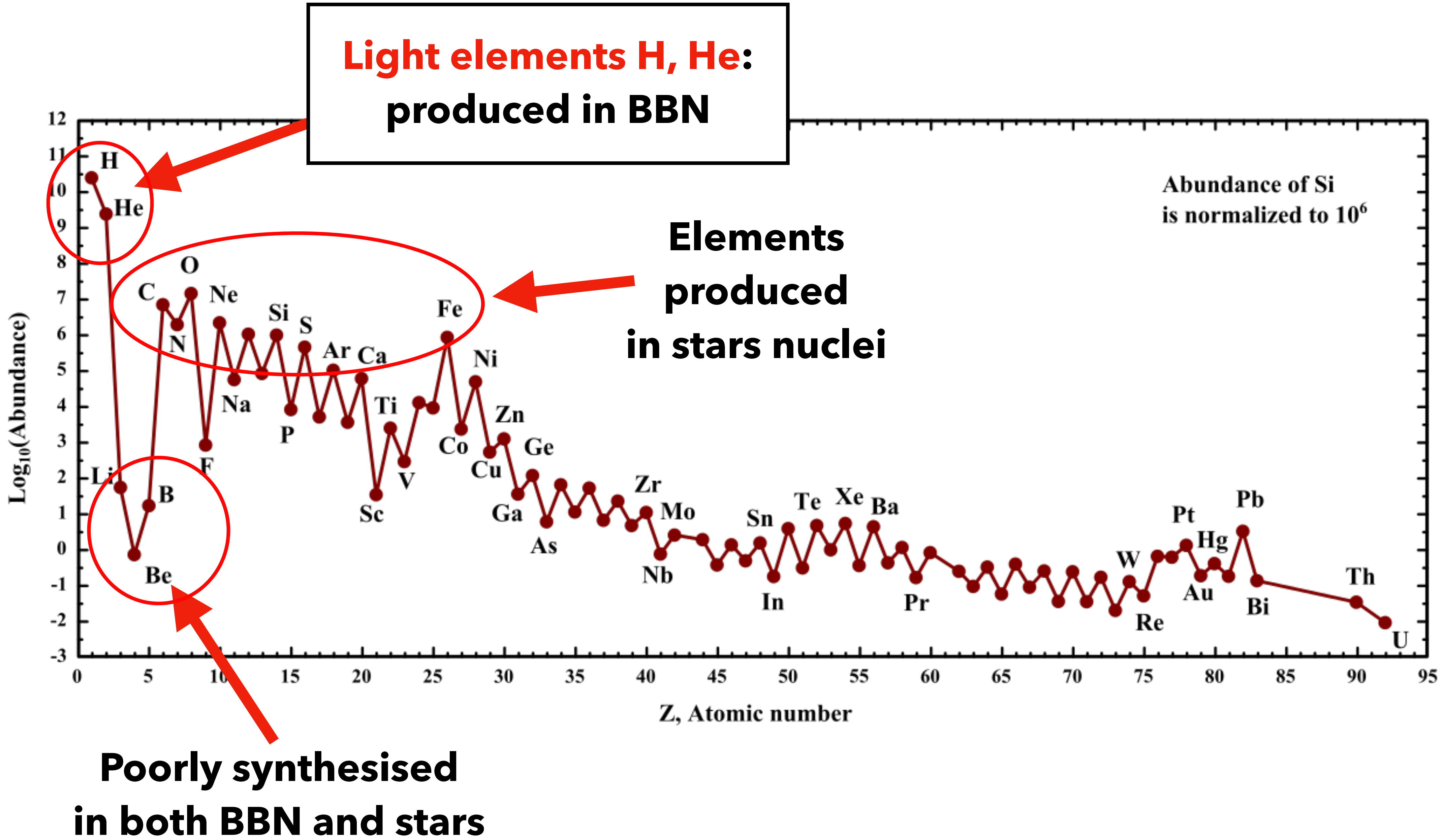
- **After 20min, the Universe is cool enough that the chain of reactions stops: there is no creation of elements heavier than Be.**

- **Predicted ratio:**

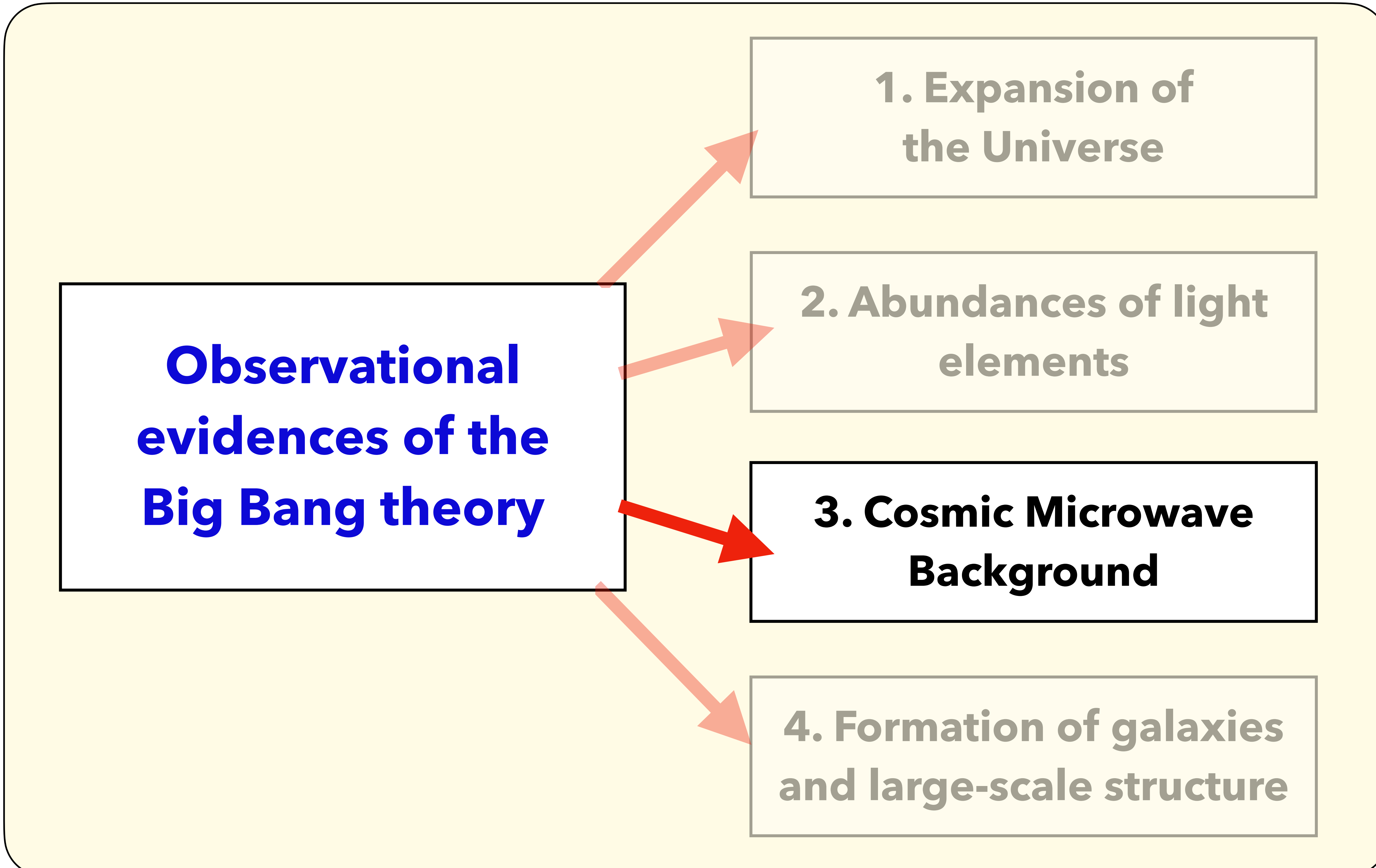
$$\frac{n_{\mathrm{He}}}{n_{\mathrm{H}}} \approx \frac{1}{4}$$

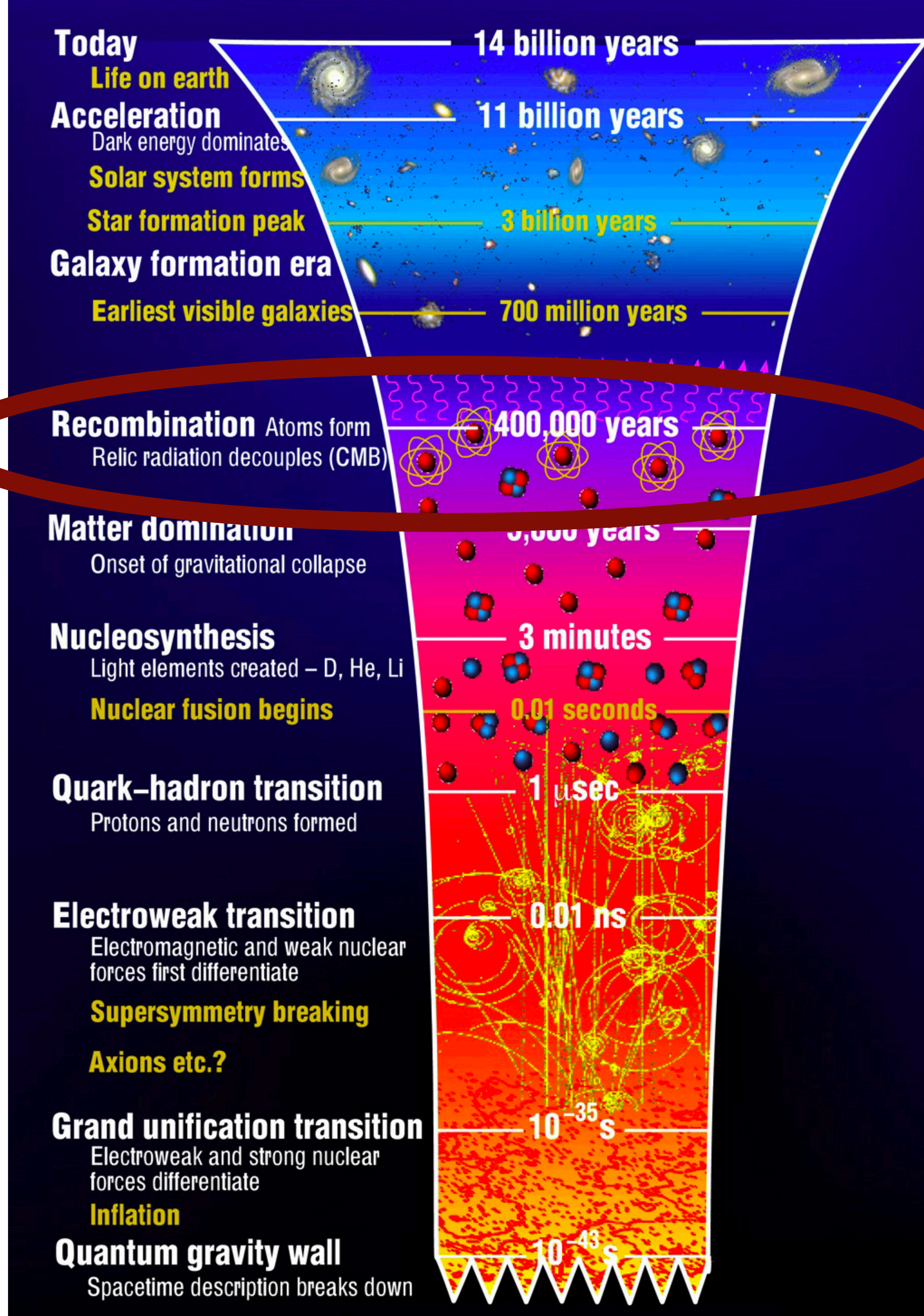
Big Bang nucleosynthesis (BBN)

Abundance of nuclear elements in the solar system



Big Bang theory





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Recombination

Recombination (t≈380000 years, T≈1eV):

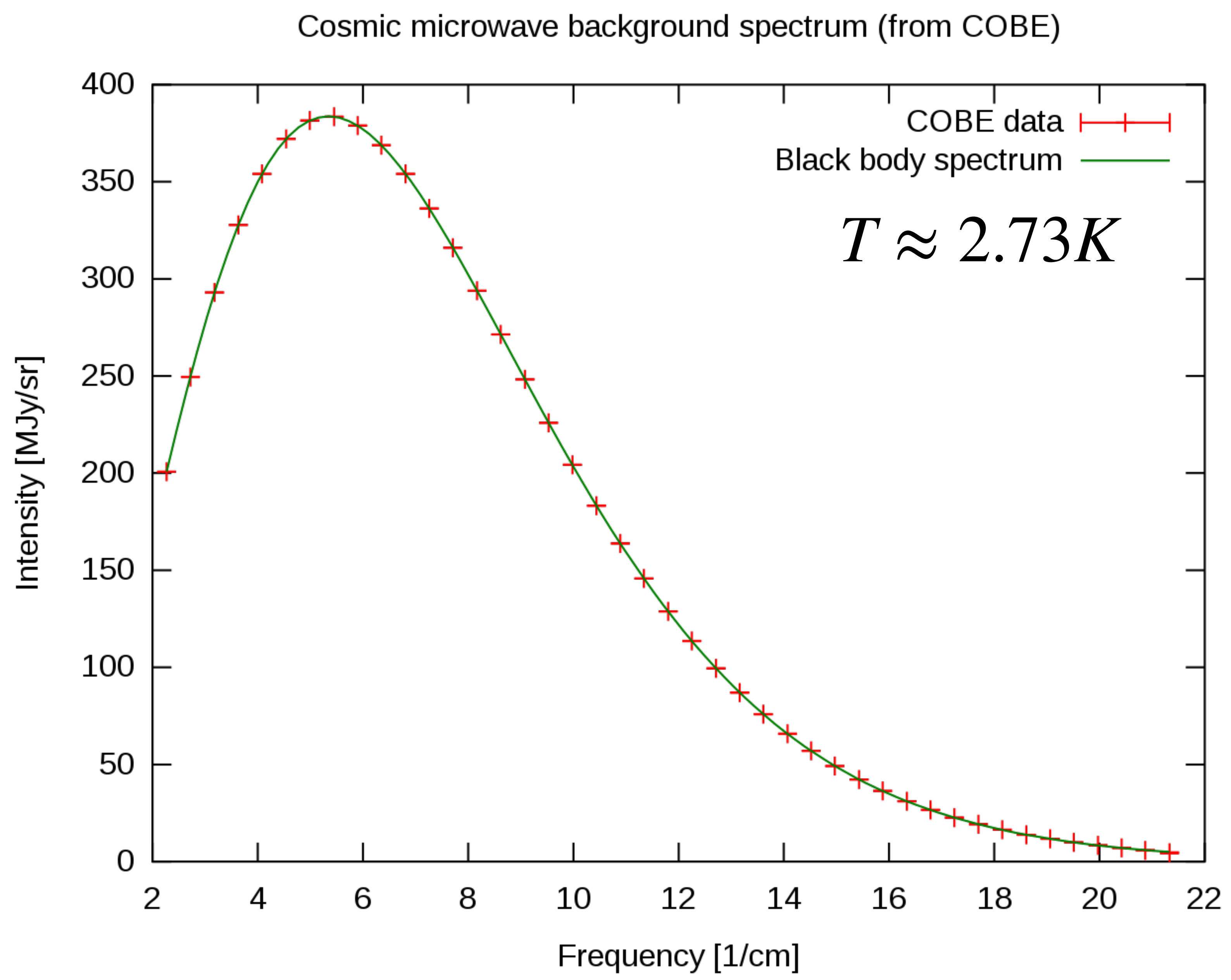
charged electrons and protons first become bound to form electrically neutral hydrogen atoms

$$e^- + p^+ \longrightarrow \mathrm{H} + \gamma$$

T≈1eV

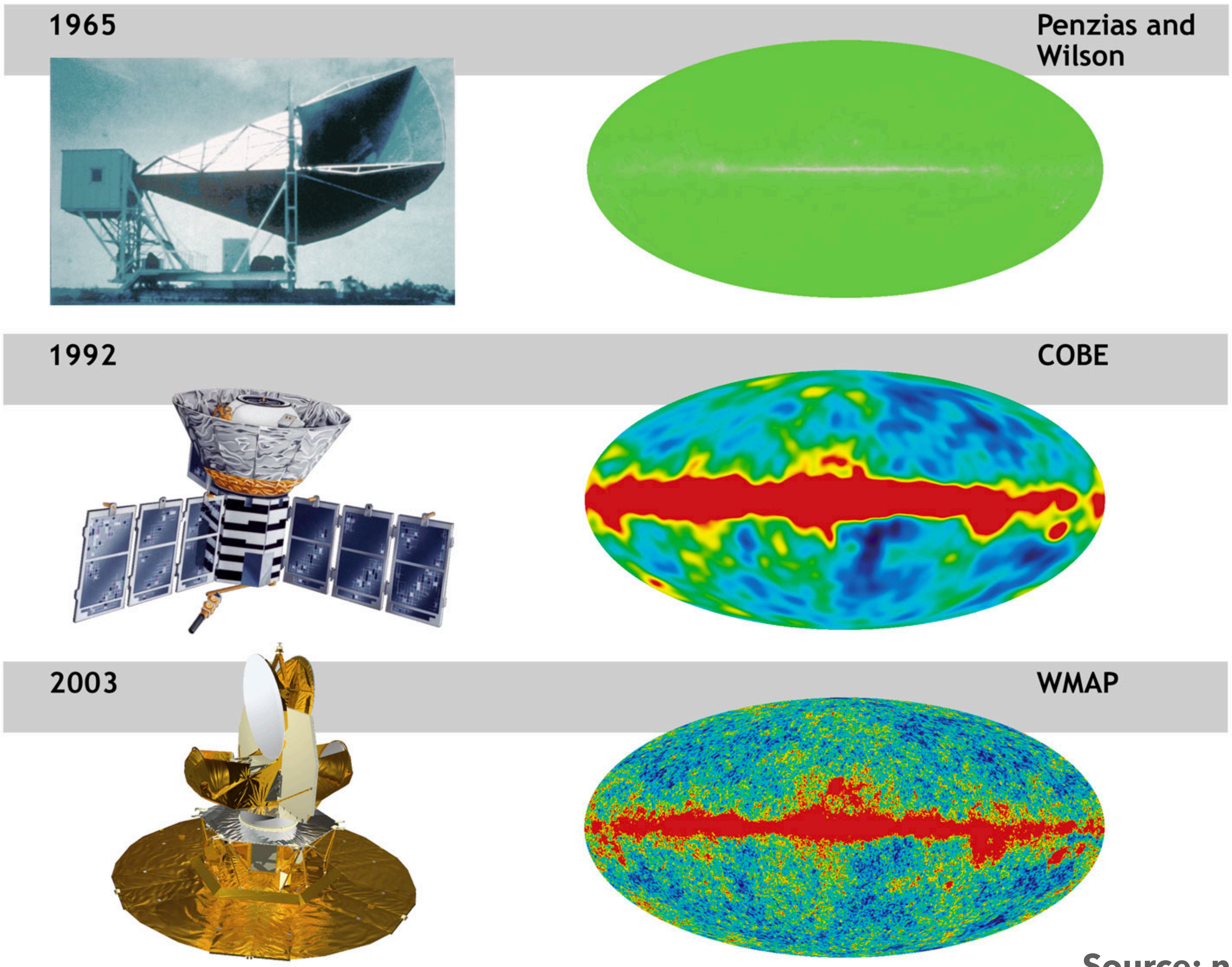
- **The photons were able to propagate large distances: the Universe becomes transparent.**
- **These photos are now detected in the microwave spectrum: the COSMIC MICROWAVE BACKGROUND (detected in 1965).**

Cosmic Microwave Background

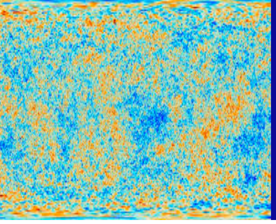


Cosmic Microwave Background

CMB anisotropies!

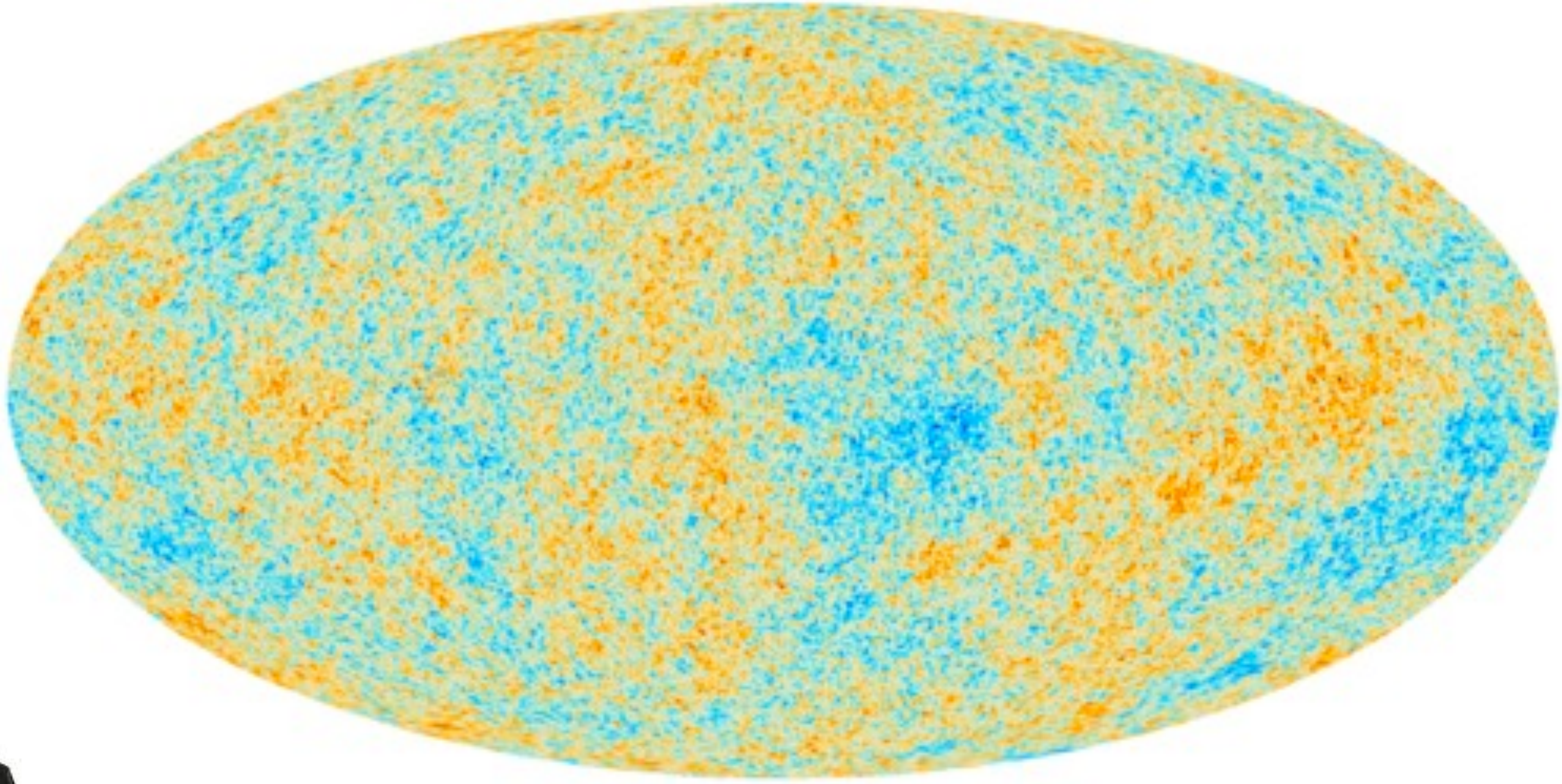


Source: nasa.gov

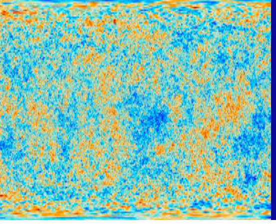


Cosmic Microwave Background

PLANCK 2018:



$$T = 2.7260 \pm 0.0013 K$$



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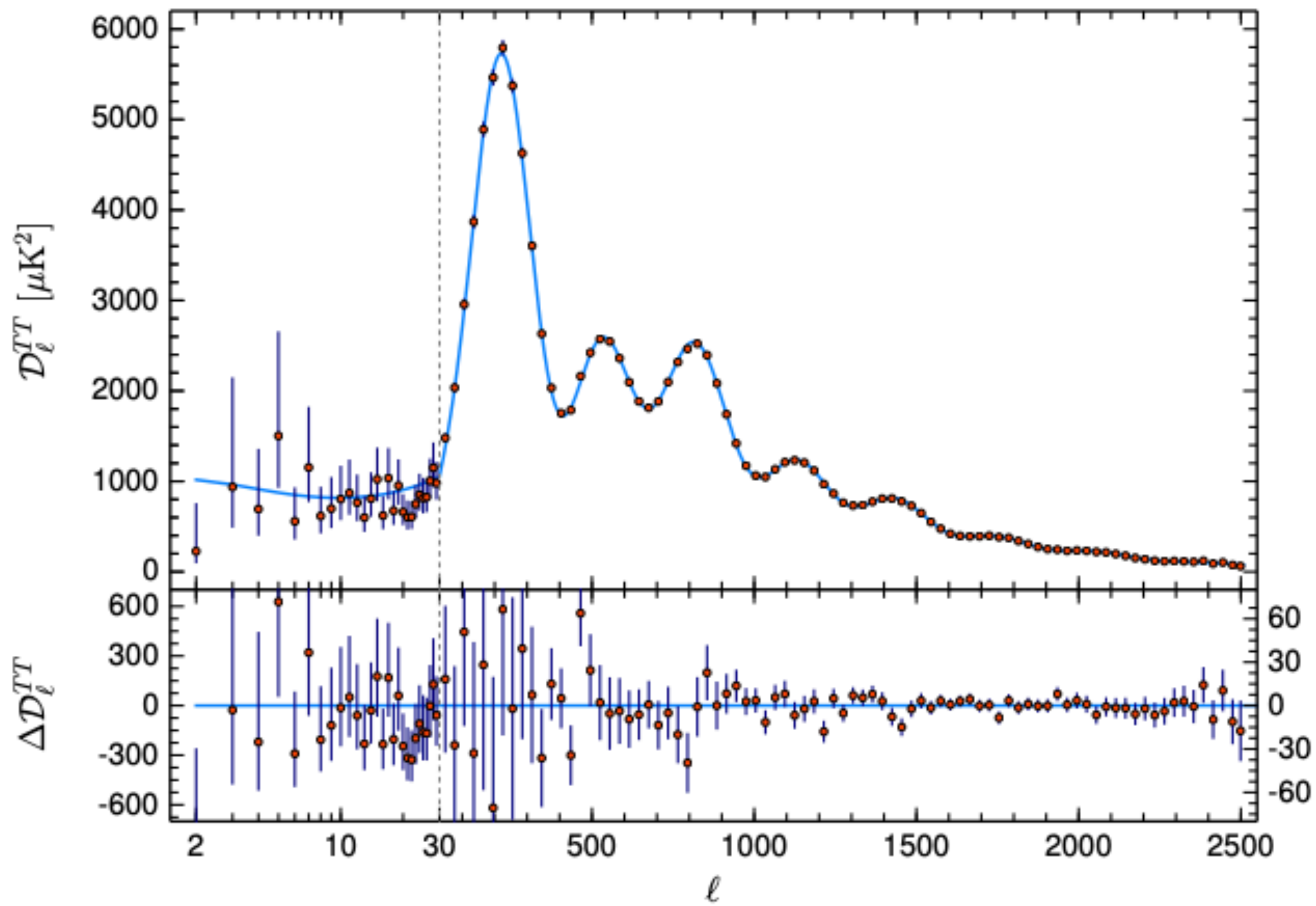

Precision cosmology

"Base ΛCDM model", parameter constraints (Planck 2018):

	Description	Symbol	Value
Indepen-dent para-meters	Physical baryon density parameter[a]	$\Omega_b h^2$	0.022 30 ±0.000 14
	Physical dark matter density parameter[a]	$\Omega_c h^2$	0.1188 ±0.0010
	Age of the universe	t_0	13.799 ±0.021 × 10^9 years
	Scalar spectral index	n_s	0.9667 ±0.0040
	Curvature fluctuation amplitude, $k_0 = 0.002\ \text{Mpc}^{-1}$	Δ_R^2	$2.441^{+0.088}_{-0.092} \times 10^{-9}$[20]
	Reionization optical depth	τ	0.066 ±0.012

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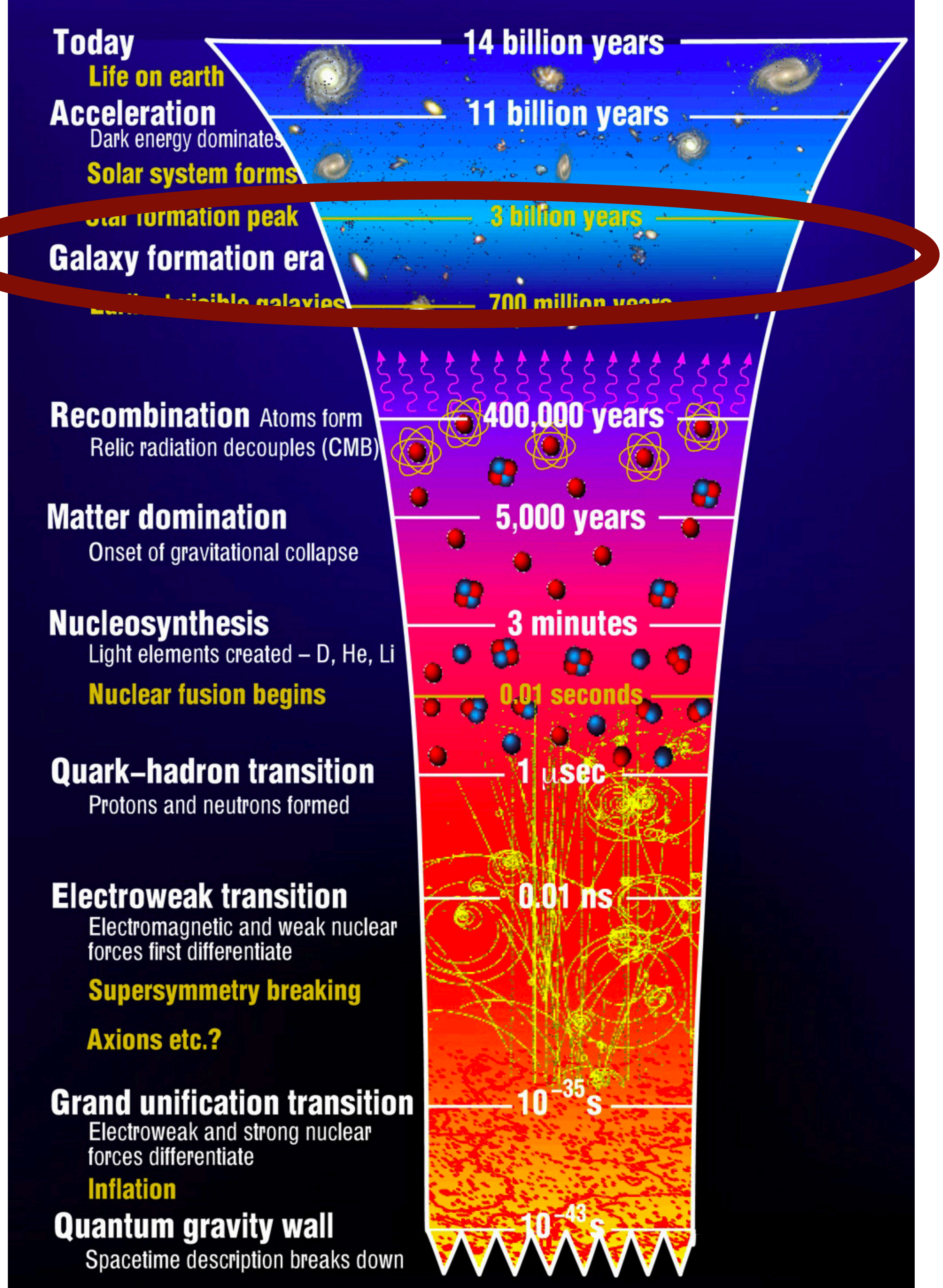

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Large scale structure

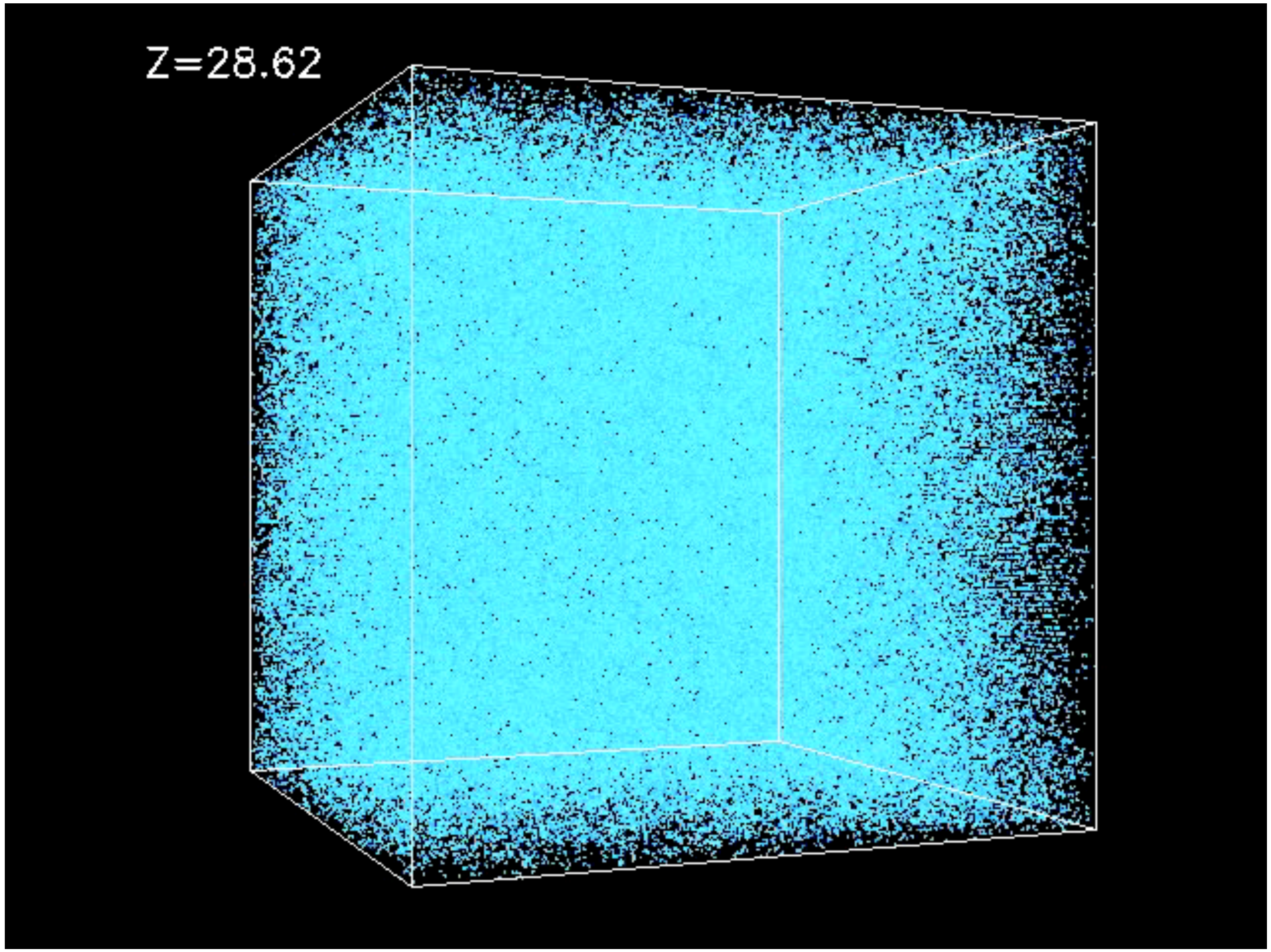
- First galaxies formed about a billion years after the Big Bang.

- After that, larger structures have been forming: galaxy clusters and superclusters.

- Populations of stars age and evolve: distant galaxies (observed as they were shortly after the Big Bang) appear very different from nearby galaxies (observed in a more recent state).

- Observations of star formation, galaxy and quasar distributions and larger structures, agree well with Big Bang simulations of the formation of structure in the universe.

Large scale structure



Source: http://cosmicweb.uchicago.edu/filaments.html

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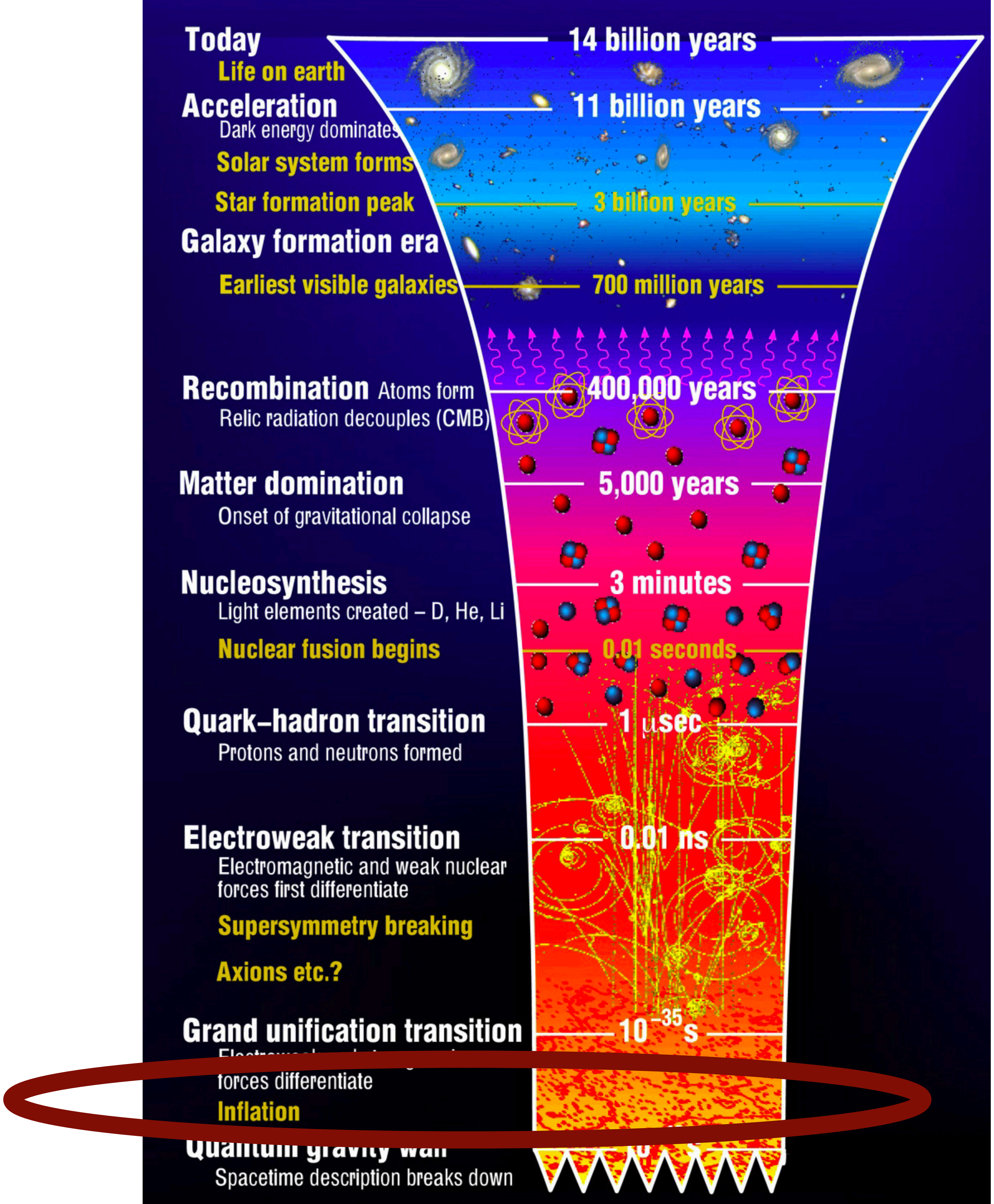

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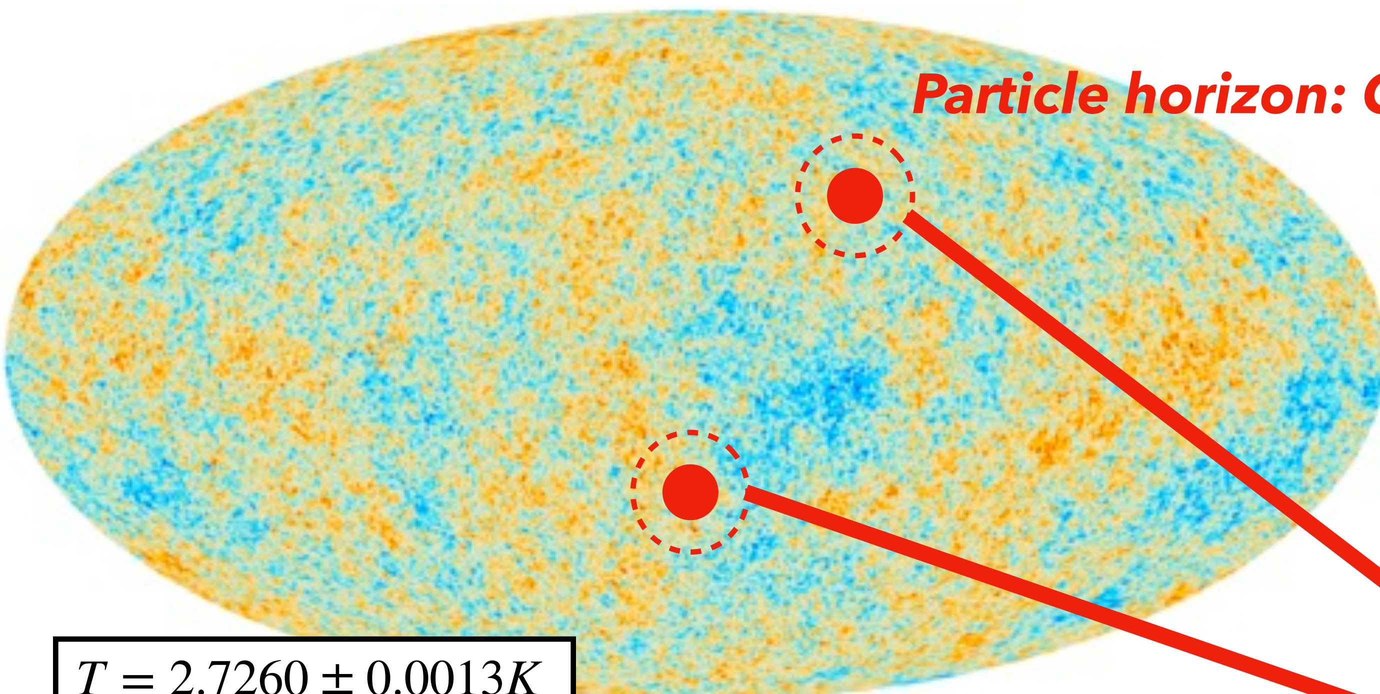
Problems of Big Bang theory

Universes dominated by matter and/or radiation → $\frac{\ddot{a}}{a} < 0$

1. The *horizon* problem

Particle horizon: Casually connected regions



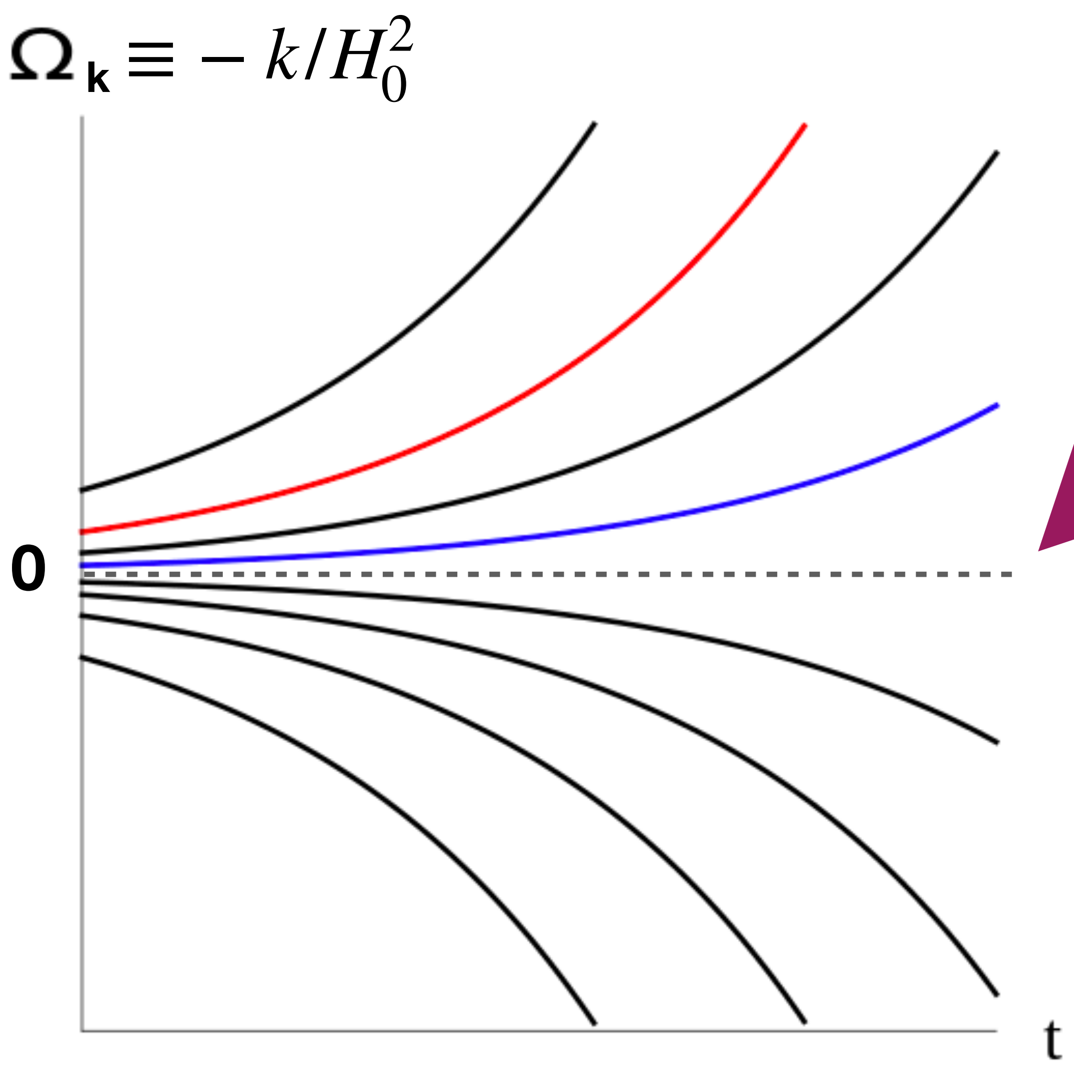
$T = 2.7260 \pm 0.0013K$

These points were never in casual contact before.
Why do they have the same temperature??

Problems of Big Bang theory

2. The *flatness* problem

$$\Omega_K = -0.011^{+0.013}_{-0.012}$$



Ω_k = 0 is a point of unstable equilibrium in the Friedmann equations

$$\Omega_k \approx 1 \qquad t \approx \text{now}$$

$$\Omega_k \approx 10^{-62} \qquad t \approx m_{\text{Pl}}$$

Inflation

PHYSICAL REVIEW D VOLUME 23, NUMBER 2 15 JANUARY 1981

Inflationary universe: A possible solution to the horizon and flatness problems

Alan H. Guth*

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 11 August 1980)

The standard model of hot big-bang cosmology requires initial conditions which are problematic in two ways: (1) The early universe is assumed to be highly homogeneous, in spite of the fact that separated regions were causally disconnected (horizon problem); and (2) the initial value of the Hubble constant must be fine tuned to extraordinary accuracy to produce a universe as flat (i.e., near critical mass density) as the one we see today (flatness problem). These problems would disappear if, in its early history, the universe supercooled to temperatures 28 or more orders of magnitude below the critical temperature for some phase transition. A huge expansion factor would then result from a period of exponential growth, and the entropy of the universe would be multiplied by a huge factor when the latent heat is released. Such a scenario is completely natural in the context of grand unified models of elementary-particle interactions. In such models, the supercooling is also relevant to the problem of monopole suppression. Unfortunately, the scenario seems to lead to some unacceptable consequences, so modifications must be sought.

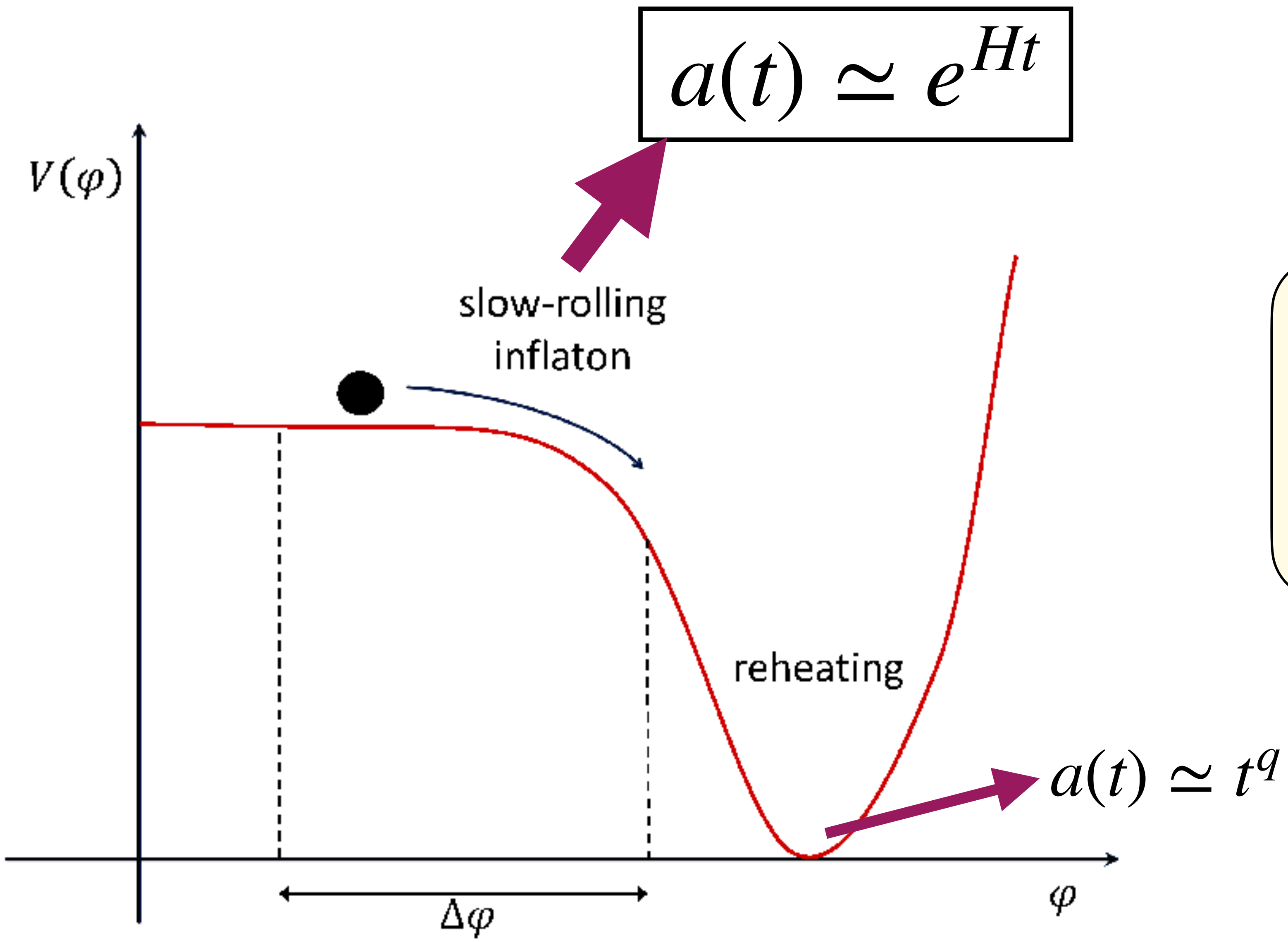
$$\frac{\ddot{a}}{a} > 0$$

The observable Universe is within the horizon

$\Omega_k = 0$ is a point of stable equilibrium

Inflation

- Inflation can be realized by a **scalar field φ** (the inflaton) with **potential energy V(φ)**, as long as V(φ) obeys certain conditions.

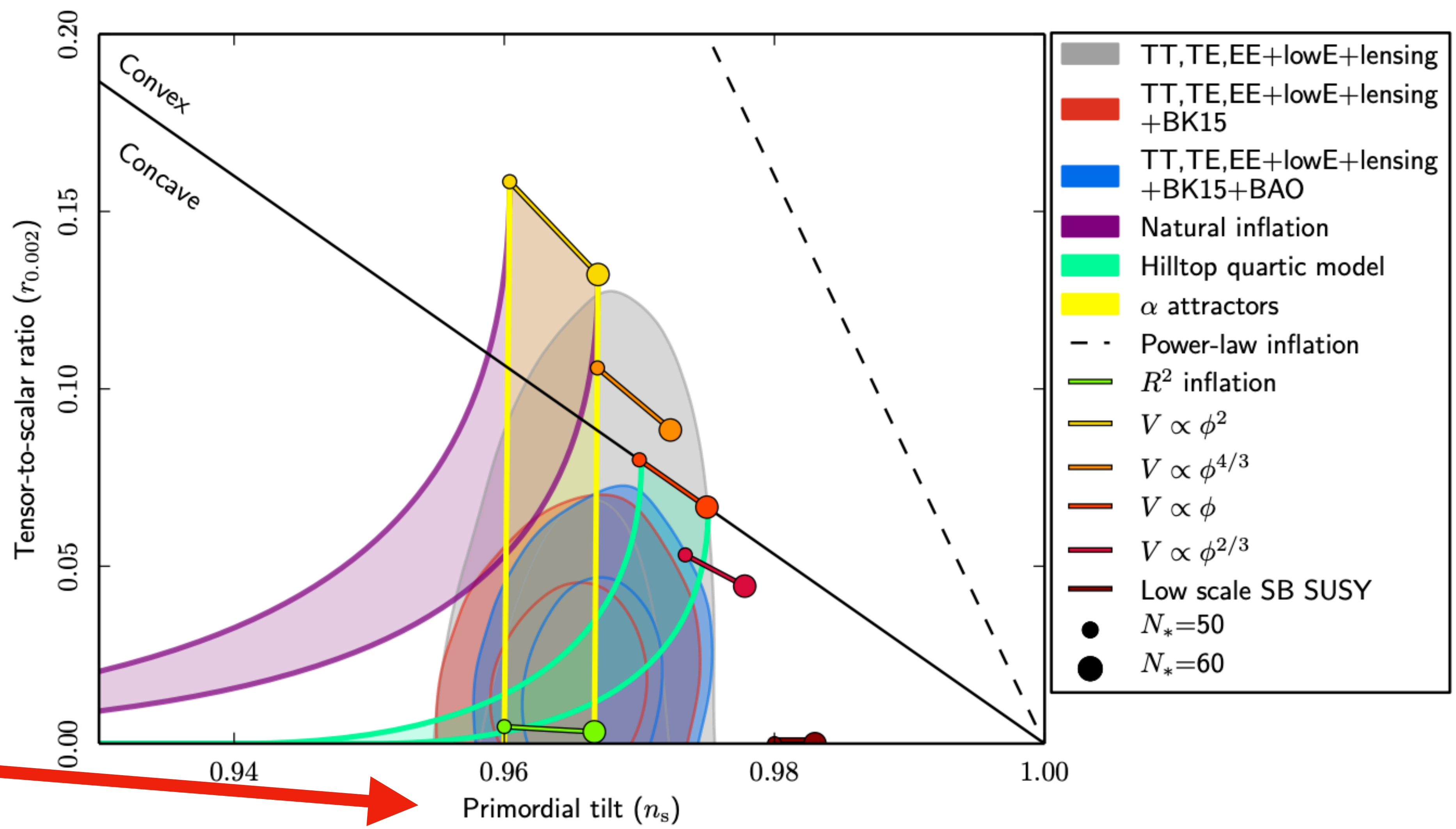


50-60 efolds of inflation solves flatness and horizon problems

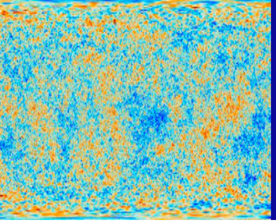
Inflation

- **Inflation generates a (quasi) scale-invariant spectrum of primordial metric perturbations: scalar and tensor fluctuations (GWs).**
- **These perturbations get imprinted in the CMB.**

Planck constraints (2018):



Different V(φ) give different predictions



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Let's start!